

Testing Quantum Gravity through Dumb Holes

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We propose a method to test the effects of quantum fluctuations on black holes by analyzing the effects of thermal fluctuations on dumb holes, the analogues for black holes. The proposal is based on the Jacobson formalism, where the Einstein field equations are viewed as thermodynamical relations, and so the quantum fluctuations are generated from the thermal fluctuations. It is well known that all approaches to quantum gravity generate logarithmic corrections to the entropy of a black hole and the coefficient of this term varies according to the different approaches to the quantum gravity. It is possible to demonstrate that such logarithmic terms are also generated from thermal fluctuations in dumb holes. In this paper, we claim that it is possible to experimentally test such corrections for dumb holes, and also obtain the correct coefficient for them. This fact can then be used to predict the effects of quantum fluctuations on realistic black holes, and so it can also be used, in principle, to experimentally test the different approaches to quantum gravity.

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The entropy of a black hole (s) scales with its area rather than its volume, $s = A/4$, where A is the area of its horizon [1, 2]. This is true not only in general relativity but also in modified theories of gravity, and so even in modified theories of gravity the leading order entropy scales with the area [3–7]. This expression for the entropy of a black hole can be obtained using a semi-classical approximation. Since any approach to quantum gravity has to agree with the semi-classical approximation at a sufficiently low energy scale, it is expected that all the approaches to quantum gravity will produce this expression for the entropy of a black hole. In fact, it has been observed that even though there are various different approaches to quantum gravity, all of them predict that the entropy of a black hole would be related to the area of the horizon as $s = A/4$. However, all these different approaches to quantum gravity also predict corrections to this relation between the area and the entropy. Non-perturbative quantum general relativity has been used to obtain such a logarithmic correction to the area-entropy law [8]. It is known that a relation exists between the density of states of a black hole and the conformal blocks of a conformal field theory in non-perturbative quantum general relativity. This relation is used to obtain the logarithmic correction to the area-entropy law. It has also been demonstrated that such a logarithmic term can be obtained using the Cardy formula for the entropy of a black hole [9]. It is also possible to analyze the exact partition function for a Bañados-Teitelboim-Zanelli (BTZ) black hole [10], and calculate the correction to the area-entropy law for such a black hole. This calculation points out that the entropy of a BTZ black hole results corrected by a logarithmic term [11]. It has also been observed that the entropy of a dilatonic black holes is also corrected by a logarithmic term [12]. The string theory corrections to the area-entropy law of a black hole have been obtained, and it has been observed that such corrections are logarithmic [13–16]. The correction term obtained from the generalized uncertainty principle is also a logarithmic term [17, 18].

In the recent work [19], the logarithmic correction applied to the dyonic charged anti-de Sitter black hole, and holographic dual Van der Waals fluid has been studied. Another kind of logarithmic correction already

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considered for the Gödel black hole [20]. Many recently, we study logarithmic correction of a sufficient small singly spinning Kerr-AdS black hole [21].

It may be noted that the Wald entropy produced by the gravity containing higher curvature terms coincides with the entropy corrections produced by generalized uncertainty principle [22]. So, we expect the modified theories of gravity with higher curvature terms would also produce such corrections. This might be because these higher curvature terms can be viewed as higher loop quantum corrections using the formalism of the effective field theories [23]. Such corrections have also been obtained using the Cardy-Verlinde formula [24, 25]. Thus, the quantum corrections from these various approaches to quantum gravity produce similar scaling behavior of the leading order corrections to the entropy.

Now, it is possible to state that all the different approaches to quantum gravity generate logarithmic corrections to the area-entropy law of a black hole. It is worth noticing that even though the leading order corrections to this area-entropy law are logarithmic corrections of the form $\ln A$, the coefficient of such a term depends on the approach to the quantum gravity. The coefficient of this term can be used as a parameter to differentiate among different approaches to the quantum gravity. Since the values of the coefficients of the logarithmic terms depend on the chosen approach to quantum gravity, it can be argued that such terms are generated from quantum fluctuations of the space-time geometry rather than from matter fields on that space-time. However, such quantum fluctuations only become important at very small scales. So, these quantum fluctuations start to become important as the size of the black hole reduces and its temperature increases. Even though it is possible to neglect the effect of quantum fluctuations for very large black holes, it is not possible to neglect such corrections for sufficiently small black hole with very high temperature.

Thus, the temperature of a black hole can be used as an indicator of the scale at which quantum fluctuations become important. This is obvious in the Jacobson formalism where the Einstein field equations are viewed as thermodynamical relations [26, 27]. In fact, in the Jacobson formalism, the Einstein equations are obtained by requiring the Clausius relation holding at the local Rindler causal horizons through each space-time point [26, 27].

Now if the Einstein equations are thermodynamical relations, then quantum fluctuations in the geometry of the space-time are generated from the thermal fluctuations in the thermodynamical system used to derive the Einstein field equations. As soon as these thermal fluctuations become relevant at high temperature, and high temperature corresponds to the small scale for black holes, the quantum fluctuations in the metric of a black hole can also be analyzed using thermal fluctuations in thermodynamics.

It has been demonstrated that the thermal fluctuations in the thermodynamics of a black hole correct the area-entropy law by a logarithmic correction term [28, 29]. In fact, the area-entropy law results modified due to thermal fluctuations as $S = s - \ln s_1/2$, where $s = A/4$ and $s_1 \sim sT^2$. Now as this logarithmic term occurs in all approaches to quantum gravity, but its coefficient depends on the specific approach to quantum gravity, we will write this entropy correction as $S = s - a \ln sT^2/2$, where we have added an arbitrary parameter a . The exact value of a can be used to discriminate among different quantum gravity models. Such an approach has been used for analyzing the effects of thermal fluctuations on the thermodynamics of an AdS black holes [30], a black Saturn [31], charged dilatonic black Saturn [32], and a modified Hayward black hole [33].

Even though this seems to be an interesting way to test models of quantum gravity using black hole thermodynamics, the problem with this approach is that it is almost impossible, from an experimental point of view, to test the effect of quantum fluctuations for real black holes. However, it is possible to test such effects for dumb holes, which are analogous black hole as solutions [34–37].

In fact, it is well known that inhomogeneous fluid flows can become supersonic and can produce acoustic analogs of black holes. Such holes are called *dumb holes*, and they can have a Hawking-like radiation of phonons with a temperature given by the gradient of the velocity at the horizon [38]. Thus, one can analyze, in principle, the effects that thermal fluctuations have on dumb holes. However, it is also possible to experimentally test the behavior of dumb holes [39], and so one can compare the analysis to the experimental data. In summary, we can experimentally verify the effects of thermal fluctuations on the thermodynamics of dumb holes.

However, since dumb holes are, in general, mathematically equivalent to the black holes, and, in the Jacobson formalism, even the real black holes are thermodynamical objects, one can use the analysis performed on dumb holes to fix the value of a for black holes. Practically, by calculating and observing the value of a from dumb holes, one can infer the correct model of quantum gravity since the coefficient of the

logarithmic term strictly depends on the approach to the quantum gravity.

In this letter we want to investigate the effects of thermal fluctuations on the thermodynamics of dumb holes in order to put constraints on the value of the coefficient of the logarithmic correction.

The construction of dumb holes is well known and its gravity dual has also been analyzed in detail [34]. The metric of the dumb hole can be written as

$$ds^2 = \sqrt{3}\mathcal{T} \left[-\left(1 - \frac{2}{3}\gamma(z)^2\right)d\tau^2 + \frac{dz^2}{3\left(1 - \frac{2}{3}\gamma(z)^2\right)} + R(z)^2 d\Omega_2^2 \right], \quad (1)$$

where \mathcal{T} is a quantity with energy dimensions. It is usually assumed to be a function of the temperature T and the chemical potential μ (or charge q). For the uncharged case ($\mu = 0$ or $q = 0$), we have $\mathcal{T} = T$. It can also be expressed in terms of a scalar field ϕ , as $\mathcal{T}^2 = -(\partial_\mu \phi)(\partial^\mu \phi)$. Here $\gamma(z)$ is a Lorentz factor defined in terms of the velocity $v(z)$,

$$\gamma(z) = \frac{1}{1 - v(z)^2}. \quad (2)$$

The curvature corresponding to this metric can be denoted as $R(z)$. It is worth noticing that $R(z) = z$ corresponds to a flat geometry. Now the expression for the acoustic Hawking temperature T_H can be written as

$$T_H = \frac{3}{4\pi} \left(\frac{dv_z}{dz} \right)_{z=z_h}. \quad (3)$$

This temperature is obtained by using the standard techniques of Euclidean quantum gravity near the horizon z_h . The only non-zero component of the velocity is v_z , and then [34]

$$v_z - v_z^3 = \frac{\Phi_s}{\mathcal{T}_\infty^3} R(z)^{-2}, \quad (4)$$

where \mathcal{T}_∞ and Φ_s denote asymptotic temperature and entropy, respectively. The gravitational dual of this model has been analyzed using the fluid-gravity correspondence [40]. The gravity dual action is obtained from type IIB supergravity with negative cosmological constant $\Lambda = -6$ [41],

$$S = \frac{1}{16\pi G} \int d^5x \left[R + 12 - F_{AB}F^{AB} - \frac{4\kappa}{3} \varepsilon^{EABCD} A_E F_{AB} F_{CD} \right], \quad (5)$$

with the Chern-Simons parameter $\kappa = 1/(2\sqrt{3})$, and G is the Newton constant in five dimension. Special case of $\kappa = 0$ corresponds to a pure Maxwell theory with no Chern-Simons type interactions. Also A_E ($E = 0, 1, 2, 3, 4$) is a $U(1)$ gauge field with strength F_{AB} . So, the resulting charged black brane solution is given by,

$$\begin{aligned} ds^2 &= -2u_\mu dx^\mu dr - r^2 f(r) u_\mu u_\nu dx^\mu dx^\nu + r^2 P_{\mu\nu} dx^\mu dx^\nu, \\ A &= \frac{\sqrt{3}Q}{2r^2} u_\mu dx^\mu, \\ f(r) &= 1 - \frac{m}{r^4} + \frac{Q^2}{r^6}, \end{aligned} \quad (6)$$

where m and Q as mass and charge parameters, and they are related to the fluid parameters by the gauge-gravity dictionary [41],

$$\begin{aligned} \epsilon &= 3\alpha m, \\ q &= \sqrt{3}\alpha Q, \end{aligned} \quad (7)$$

where $\alpha \equiv (16\pi G)^{-1}$. From the fluid side of this duality, the first law of thermodynamics can be expressed as

$$d\epsilon = Tds + \mu dq, \quad (8)$$

where s denotes uncorrected entropy, and ϵ is the energy density which is related to the pressure via $\epsilon = 3p$. Now, by using the fluid-gravity correspondence, it can be demonstrated that [34]

$$\begin{aligned}\epsilon &= 3\alpha \left(\frac{s}{4\pi\alpha}\right)^{\frac{4}{3}} + \frac{q^2}{\alpha} \left(\frac{s}{4\pi\alpha}\right)^{-\frac{2}{3}}, \\ T &= \frac{1}{\pi} \left(\frac{s}{4\pi\alpha}\right)^{\frac{1}{3}} \left(1 - \frac{8\pi^2 q^2}{3s^2}\right), \\ \mu &= \frac{2q}{\alpha} \left(\frac{s}{4\pi\alpha}\right)^{-\frac{2}{3}}.\end{aligned}\tag{9}$$

The effect of thermal fluctuations for this system can be analyzed by relating it to a conformal field theory [28], and using the modular invariance of the partition function of the conformal field theory [9]. Now if $S = A\beta_\kappa^l + B\beta_\kappa^{-j}$, where $\beta_\kappa = (\kappa_B T)^{-1}$, and $l, j, A, B > 0$, then the relation between the corrected entropy S and the original entropy s , can be written as [28, 32]

$$S = s - \frac{a}{2} \ln |sT^2| + \dots,\tag{10}$$

where the variable a is again introduced to parameterize the effect of thermal fluctuations on the thermodynamics of dumb hole. The original entropy and partition function are related to each other by the following differential equation,

$$s = X + T \frac{dX}{dT}.\tag{11}$$

Here $X \equiv \kappa_B \ln Z$ has been introduced to simplify relations, where Z is the partition function. Now we can obtain partition function from

$$X = \frac{1}{T} \left[\int s dT + c \right],\tag{12}$$

where c is an integration constant. Thus, the partition function can be obtained if we know the temperature dependence of the entropy. In the case of entropy corrected by thermal fluctuation, we can use Eqs. (10) and (11) to obtain the modified partition function,

$$X = \frac{1}{T} \left[\int g(S) dT + C \right],\tag{13}$$

where C is another integration constant and,

$$g(S) = -\frac{a}{2} LW \left(-\frac{2e^{-\frac{2S}{a}}}{aT^2} \right),\tag{14}$$

where $g(S)$ is obtained from Eq. (10) as the Lambert W function LW . In the case of $a = 0$ where $g(S) = s$ we have ordinary thermodynamics. The partition function given by Eq. (13) can be used to obtain thermodynamics information about this system. Thus, we write internal energy as

$$U = T^2 \frac{dX}{dT},\tag{15}$$

and then specific heat,

$$C_v = \frac{dU}{dT}.\tag{16}$$

Obtaining general analytical solutions for Eq. (13) is very difficult. In this paper, we restrict the analysis to uncharged dumb holes ($q = 0$). In this case, it is possible to obtain analytical results. Now, $s = 4\alpha\pi^4 T^3$ is obtained using Eqs. (9). Therefore, entropy corrected by thermal fluctuations is given by,

$$S = 4\alpha\pi^4 T^3 - \frac{a}{2} \ln 4\alpha\pi^4 T^3.\tag{17}$$

The function $g(S)$ is given by Eq. (14), and it goes to infinity at zero temperature. For finite temperature in the range $0 < T < \infty$, it behaves as follow,

$$g(S) = \frac{e^{-\frac{2S}{a}}}{T^2} + \mathcal{O}\left(\frac{1}{T^4}\right). \quad (18)$$

However, in the zero-temperature limit $T \rightarrow 0$, we have $\frac{1}{T^2} \rightarrow \infty$. The thermal fluctuations vanishes for $a = 0$. Furthermore, in the limit $T \rightarrow \infty$, it is $g \rightarrow \text{constant}$, with infinitesimal values. Hence $\kappa_B \ln Z \approx c_1 + \frac{c_2}{T}$. Now using Eqs. (17) and (18), we obtain

$$g(S) \approx 4\alpha\pi^4 T^3 e^{-\frac{8\alpha}{a}\pi^4 T^3}. \quad (19)$$

Therefore, by using Eqs. (19) in (13), we also obtain

$$X = AT^{-\frac{1}{2}} e^{-\frac{4\alpha\pi^4}{a}T^3} WM\left(\frac{1}{6}, \frac{2}{3}, \frac{8\alpha\pi^4}{a}T^3\right) + \frac{C}{T}, \quad (20)$$

where $A \equiv \frac{\sqrt{2}a^{\frac{7}{6}}}{16\pi^{\frac{2}{3}}\alpha^{\frac{1}{6}}}$, and $WM\left(\frac{1}{6}, \frac{2}{3}, h\right)$ is Whittaker function M with $h \equiv \frac{8\alpha\pi^4}{a}T^3$. Before analyzing the general solution, using expression (20), we will use approximations for some special cases. At the first order approximation, we obtain,

$$Z = e^{B_1 T^3 + \frac{C}{T}}, \quad (21)$$

where B_1 is a constant which depends on α . Thus, we get

$$\begin{aligned} U &= 3B_1 T^4 - C, \\ C_v &= 12B_1 T^3, \end{aligned} \quad (22)$$

which are derived from the original entropy given in Eq. (12). Now the effects of thermal fluctuations can be obtained by using higher order corrections to the Whittaker function M given in Eq. (20). At the second order approximation, we obtain

$$Z = e^{B_1 T^3 + B_2 T^6 + \frac{C}{T}}, \quad (23)$$

where B_1 and B_2 are constants depending on α . We can write

$$\begin{aligned} U &= 3B_1 T^4 + 6B_2 T^7 - C, \\ C_v &= 12B_1 T^3 + 42B_2 T^6. \end{aligned} \quad (24)$$

Hence, we obtained the effect of thermal fluctuations parameterized by the coefficients $B_{1,2}$. It is clear that, by the increasing temperature, these thermal fluctuations become important, while, at lower temperatures, they can be neglected. These are only approximate solutions. We need now the general case. It is given by Eq. (20). We can plot U and C_v in terms of T^3 (see Fig. 1).

It is easy to see that the internal energy decreases at infinitesimal temperature where thermal fluctuations can be neglected. However, if these thermal fluctuations are not neglected, then the internal energy increases till a critical temperature T_c . Thus, the thermal fluctuations occur at $T_{min} \leq T \leq T_c$. As we can see from the second plot, system will become unstable at $T \geq T_c$, and we obtain $C_v = 0$ at very high temperature.

In this letter, we have analyzed the effect of thermal fluctuations on the thermodynamics of acoustic dumb holes occurring in inhomogeneous supersonic fluids. It was observed that these thermal fluctuations correct the entropy of a dumb hole by a logarithmic term. We then argued that since it is well known that all approaches to quantum gravity generate logarithmic corrections to the entropy of a black hole, and the coefficient of this term depends on the quantum gravity approach, the coefficient of this logarithmic can be used as a tool to test the correct quantum gravity model. It is important to stress that the Einstein equations can be viewed as thermodynamical relations in the Jacobson formalism, and so the quantum

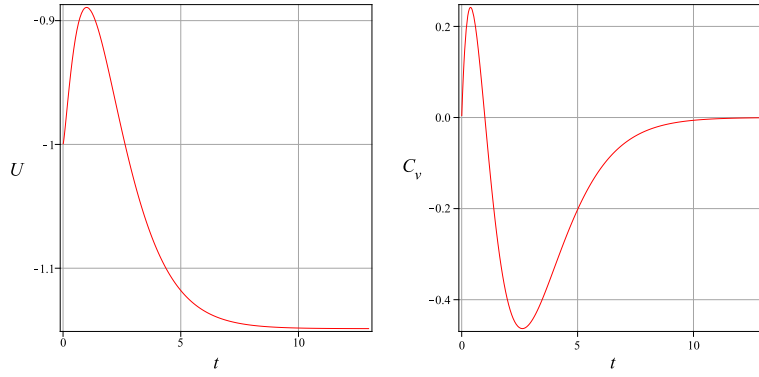


FIG. 1: Internal energy and Specific heat in terms of $t \equiv T^3$ with $C = a = 1$ and $G = \frac{\pi^3}{2}$.

fluctuations can be obtained from thermal fluctuations in this formalism. Here, we proposed that the effect of thermal fluctuations on the thermodynamics of dumb holes can be used to predict the behavior of quantum fluctuations on real black holes in the Jacobson formalism. These analog effects could be experimentally measured for a dumb hole, and thus we can experimentally obtain, in principle, the exact value of the coefficient of logarithmic correction for dumb holes. Thus, we can expect to predict the effect of thermal fluctuations for real black holes, if we know its behavior of dumb holes. However, as the coefficient of the logarithmic term depends on the approach to quantum gravity, we can also use this analysis, in general, to test the correct approach to quantum gravity.

For the future works it is interesting to apply our method to some important kinds of black holes like a new regular black hole [42], Horava-Lifshitz black hole [43], Myers-Perry black hole [44] or STU black hole [45, 46]. It would also be interesting to analyze the effect of logarithmic term on the holographic heat engines [47] dual to the mentioned black holes. Finally it is interesting to consider charged dumb hole and study the effect of thermal fluctuations on the charged dumb hole.

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